Measuring the charge-to-mass ratio of the electron using relativistic dynamics

Lily Sun*
MIT Department of Physics
(Dated: April 17, 2026)

We measured the charge-to-mass ratio of the electron using electrons from a ^{90}Sr/^{90}Y source in a uniform magnetic field. The magnetic field selects electrons with a specific momentum p, and their velocities v are measured using a crossed-field velocity selector. The electrons' p-v relationships follow relativistic rather than classical dynamics, and we obtain a value of $e/m = (1.31 \pm 0.08_{\text{sys}} \pm 0.06_{\text{stat}}) \times 10^{11}$ C/kg. We discuss possible sources of the $\sim 24\%$ discrepancy from the accepted value of 1.76×10^{11} C/kg.

I. INTRODUCTION

The charge-to-mass ratio e/m of the electron was one of the earliest measured quantities associated with a subatomic particle. In 1897, Thomson [1] measured the charge-to-mass ratio of cathode-ray particles using electric and magnetic deflection. Although he did not yet know these particles were "electrons", his results showed that they were far lighter than any known atom. Later in 1901, Kaufmann [2] investigated how this charge-to-mass ratio depended on speed using faster electrons from radioactive decay, and found that it decreased with speed. At the time, this effect was interpreted in terms of theories of electromagnetic mass; today we understand this to be a consequence of Einstein's theory of special relativity.

In this experiment, we test whether high-energy electrons emitted in radioactive decay obey classical or relativistic dynamics. By using a magnetic field to determine the electron momentum and a velocity selector to determine its velocity, we can compare the measured relation with the predictions of classical and relativistic mechanics. This also allows us to determine the electron charge-to-mass ratio, e/m.

II. THEORY

To determine if high-energy electrons follow classical or relativistic dynamics, we will determine the relationship between the momentum p and velocity v of electrons.

A schematic of the setup is shown in Figure 1. A spherical electromagnet creates a uniform magnetic field $\vec{B}$ in its cross section pointing into the page. A ^{90}Sr/^{90}Y source is placed at a radius $\rho = 20.3 \pm 0.2$ cm from the center and emits electrons with a range of velocities, from the $^{90}\text{Sr}\rightarrow^{90}\text{Y}$ and $^{90}\text{Y}\rightarrow^{90}\text{Zr}$ decays that emit electrons of maximum energies 0.546 MeV and 2.27 MeV respectively. Due to the Lorentz force $\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = q(\vec{E} + \vec{v} \times \vec{B})$, an electron travelling at velocity $\vec{v}$ would experience a magnetic force $F_B = evB$ perpendicular to its velocity, causing it to travel in a circle at a constant speed.

Electrons with different v experience different F_B and thus travel along different circular paths. However, there is a barrier in the middle with only a narrow slit at radius ρ which blocks all electrons except those travelling along the path of radius ρ. The VS is also placed at radius ρ to only be able to detect electrons following that path. This circular path is described by $\left|\frac{\mathrm{d}\vec{p}}{\mathrm{d}t}\right| = \omega p$ since the magnitude of electron momentum $\vec{p}$ does not change but its direction changes at an angular speed $\omega = v/\rho$. Therefore, by writing $\left|\frac{\mathrm{d}\vec{p}}{\mathrm{d}t}\right| = evB = vp/\rho$ we find that the momentum of electrons arriving at the VS is $p = \rho e B$.

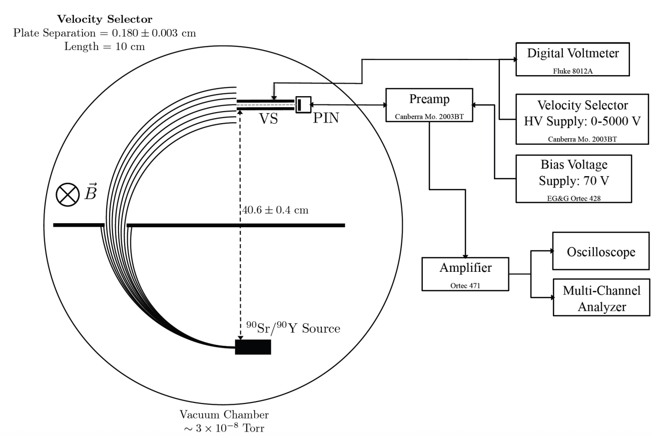

FIG. 1. **Schematic of setup.** Electrons travel in a circular path due to the magnetic field, so only those with a specific momentum p can reach the VS. Figure from lab manual.

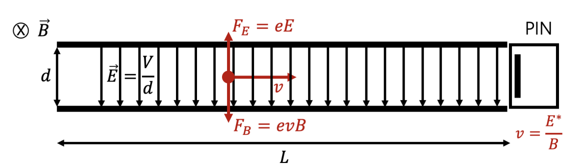

FIG. 2. **Schematic of velocity selector (VS).** Only electrons with a specific velocity v pass through and are detected.

The velocity selector (VS) will now measure the v of these electrons. The VS consists of 2 parallel plates of length $L = 10$ cm with separation $d = 0.180 \pm 0.003$

* xqsun@mit.edu

cm. A voltage V is applied between plates, creating a uniform electric field E in the direction such that the electric force on electrons $F_E = eE$ opposes the magnetic force $F_B = evB$. Only if these two forces are equal $eE = evB$ can the electron travel straight through and be detected at the PIN diode, therefore, the VS selects electrons with velocity $v = E/B$. To measure the velocity of electrons we thus have to vary V until we create the E^* that balances the F_B on those electrons. Note however that since the VS has finite width d, a slightly different $E \neq E^* = vB$ can still lead to electrons being detected as electron paths curve but still hit the PIN diode. Therefore, we have to find the E^* that leads to *maximum* counts of electrons detected and then find $v = E^*/B$.

Once the electron hits the PIN diode, the signal gets amplified through a signal chain (Figure 1) and is registered at the multi-channel analyzer (MCA) as a count in a certain channel (where the channel number is proportional to the energy of that electron). For our experiment, we are not concerned with the energy of the electron and only care about the total count rate.

Once we know $p = \rho eB$ and $v = E^*/B$ of the electron, we can determine if their relationship follows classical or relativistic dynamics.

1. In classical dynamics, $p = mv$. We define $\beta \equiv v/c = E^*/Bc$ and obtain $\rho eB = m\beta c \implies B/\beta = mc/e\rho$. A plot of B against β would thus be a straight line with slope $mc/e\rho$ giving us e/m.

2. In relativistic dynamics, $p = \gamma mv$ where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$. Therefore, $\rho eB = \gamma m\beta c \implies B/\gamma\beta = mc/e\rho$. The plot of B against $\gamma\beta$ (rather than β) would then be a straight line with a slope $mc/e\rho$ giving us e/m.

Our procedure is thus as follows:

1. Fix a value of B by setting the current in the electromagnet. Measure B using a Hall effect probe placed directly above the vertical slit. Allow the electromagnet to heat up while adjusting B to the desired value until it does not fluctuate.

2. Set V to the value predicted by relativistic dynamics. Record counts for ~ 100 s.

3. Keeping B fixed, vary V by $\sim \pm 15\%$ for at least 5 points, capturing counts at each V. Plot the count rates against V live during data collection to ensure there are points on both sides of the V^*.

4. Repeat steps 1-3 for different values of B. This allows us to plot B against β or $\gamma\beta$.

III. RESULTS

Figure 3 shows an example of an MCA spectrum for $B = 90$ G, $V = 3.051$ kV. We see a peak corresponding to electrons from the ^{90}Sr/^{90}Y source, but these electrons do not have exactly the same energies (velocities) as the VS is imperfect. Therefore, there is a range of channels whose counts must be included.

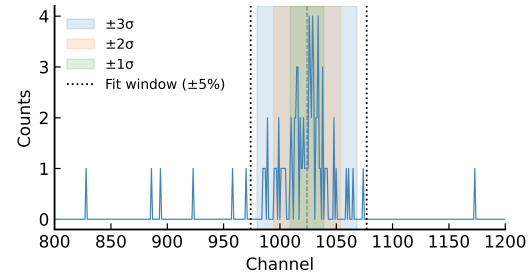

FIG. 3. **Example spectrum detected by MCA.** We fit a Gaussian around the peak count channel and sum the number of electrons within those channels.

To determine the total count N, we thus fit a Gaussian to the channels within $\pm C\% = \pm 5\%$ of the peak channel, and then count the number of electrons within $\pm M\sigma = \pm 2\sigma$. This arbitrary choice is discussed more in Section IV. The error in N is $\sqrt{N}$ as this is a Poisson process. We divide N by the live time of the MCA to get the count rate.

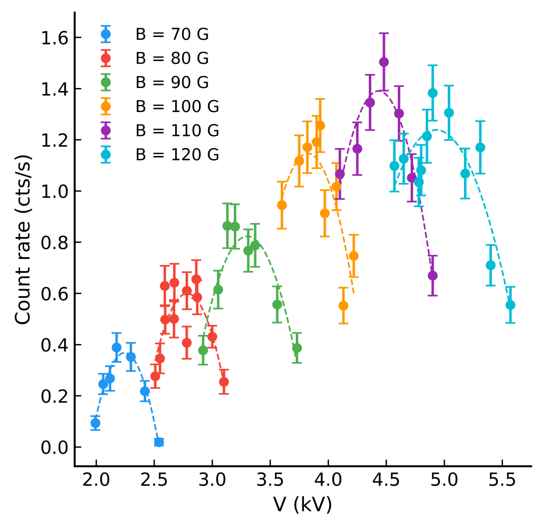

FIG. 4. **Count rate vs. voltage of VS, at different** B**.** We fit a parabola to determine V^* and hence E^*.

We used 6 values of B and took at least 5 points of V for each, shown in Figure 4. To determine V^* and thus E^*, we fit a parabola to the count rate against V plot for each B, as the maximum of an arbitrary function is approximately a parabola to 2nd order (we also discuss this choice in Section IV). The error in E^* is obtained from the error in the fitted parameters. We note that the maximum count rate varies with B as the source emits a different number of electrons with $p = \rho eB$, however, we do not care about the value of this maximum count rate, only the E^* at which it occurs.

Using those (B, β) values we can plot B against β (Figure 5) and B against $\gamma\beta$ (Figure 6). We see that B against $\gamma\beta$ is better described by a straight line ($\chi^2/\text{dof} = 1.65$) than B against β ($\chi^2/\text{dof} = 7.26$), showing that electrons follow relativistic dynamics.

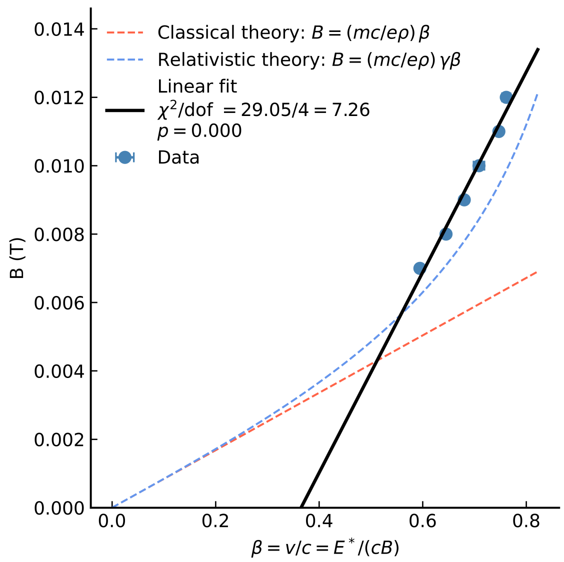

FIG. 5. B **against** β**, which classical dynamics predicts to be a straight line.**

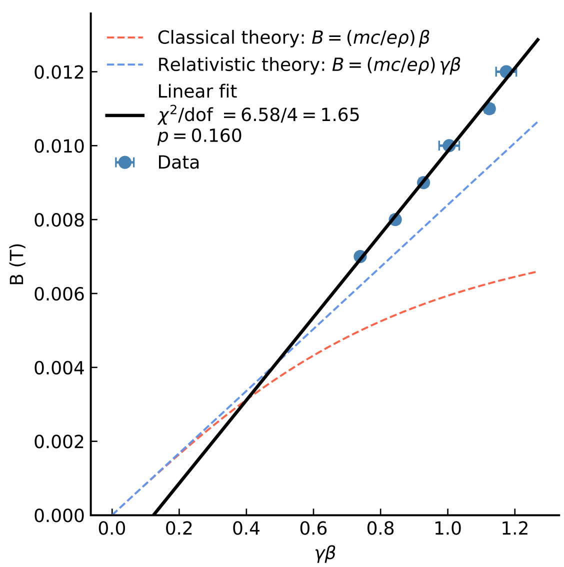

FIG. 6. B **against** $\gamma\beta$**, which relativistic dynamics predicts to be a straight line.** The linear fit is better than in the classical case.

Indeed, using our linear fit slopes $mc/e\rho$ assuming each classical and relativistic dynamics, we can obtain e/m (Table I) and see that the relativistic value is much closer to the accepted value.

However, our results still systematically deviate from relativistic theory, such that the accepted value is only

	Classical	**Relativistic**	**Accepted**
e/m (10^{11} C/kg)	0.504 ± 0.037	1.31 ± 0.06	1.76
% from accepted	-71.3%	-25.3%	-
Accepted within	34σ	7.9σ	-

TABLE I. **Values of** e/m **assuming classical and relativistic dynamics,** compared with the accepted value. Relativistic dynamics gives a closer value.

within 7.9σ of our value. This points to a systematic error in our data, which, by observing how our points consistently lie above the relativistic theory line in Figure 6, could be due to 1. an overestimation of B or 2. an underestimation of E^* and thus β. We examine possible sources of this large $\sim 24\%$ error in the next section.

IV. ANALYSIS OF ERRORS

Systematic error due to choice of C **and** M **in counting** N**.** In determining the count rate N, we fitted a Gaussian to an arbitrary channel bandwidth C and use an arbitrary width M for counts. We therefore sweep a grid of C and M to repeat our analysis showing that they do not strongly affect e/m (Figure 7).

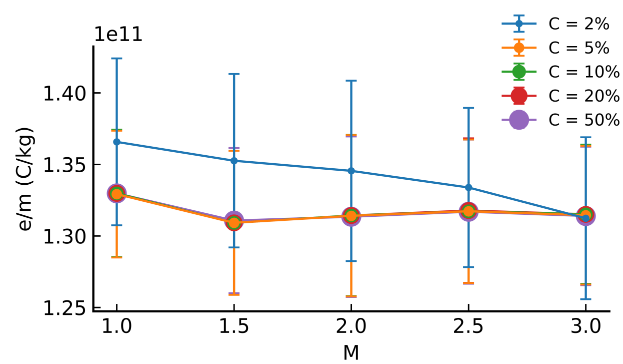

FIG. 7. **Values of** e/m **using different choices of channel width and Gaussian width for counting** N**.** These choices do not change e/m much.

Systematic error due to choice of parabola in determining E^***.** Similarly, we made the choice of using a parabola fit on count rate vs. V to determine E^*. We repeat the analysis using a Gaussian fit instead, to get new values of e/m. We find that this choice also does not cause a significant systematic error (Table II).

	Gaussian	**Parabola**	**Accepted**
e/m (10^{11} C/kg)	1.33 ± 0.05	1.31 ± 0.06	1.76
% from accepted	-24.1%	-25.3%	-
Accepted within	7.7σ	7.9σ	-

TABLE II. **Values of** e/m **using Gaussian or parabola fits to determine** E^***.** This choice does not affect e/m much.

Systematic error due to magnetic field. The magnetic field could be non-uniform, such that our measurement of B was not the true B experienced by the electrons. We measured the magnetic field at different points (Figure 8). B_0 was measured at our original point above the slit. For each azimuth (positive = towards VS), radius, and height (relative to original point), we scan in both directions so we measure each point twice to check for any systematic drift over time. One possible error is that the setup was below the equatorial plane (in the vacuum) while the magnetic field was measured at the equatorial plane, but this deviation is at most $\sim 1\%$. We see the most variation radially and thus estimate the error due to magnetic field non-uniformity to be $\sim 3\%$.

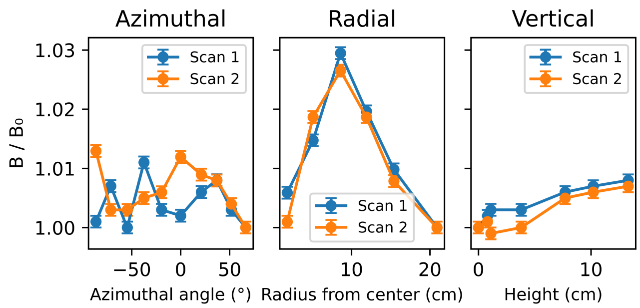

FIG. 8. **Ratio of B measured at different points with B_0 measured at original point**. The magnetic field is slightly non-uniform.

Summary of systematic errors. We put together all our known systematic errors to obtain a total systematic error of 6.42% (Table III), for a final value of $e/m = (1.31 \pm 0.08_{\text{sys}} \pm 0.06_{\text{ stat}}) \times 10^{11}$ C/kg. The uncertainties do not explain the $\sim 24\%$ deviation from the accepted 1.76×10^{11} C/kg, suggesting an unknown source of error. We think the most likely explanation is the miscalibration of the Hall probe—while we had zeroed it and checked it against a known 100 G source, we did notice some drift in the magnetic field readings when no field was present before and after the experiment. Further experiments would use multiple Hall probes or magnetic field measurements to reduce this error.

Source (X)	$\delta X/X$	$\delta(e/m)/(e/m)$
d	1.67%	1.67%
ρ	0.99%	0.99%
B	2.95%	5.90%
Choice of C/M	-	0.53%
Gaussian/Parabola	-	1.53%
Total systematic		**6.42%**

TABLE III. **Sources of systematic error.**

V. DISCUSSION OF VS

Finally, we discuss if the observed spread in V that give non-zero electron counts is reasonably close to the spread in V predicted by the finite size of the VS. We can derive the theoretical range of velocities $v^* \pm \Delta v$ accepted by the velocity detector as follows.

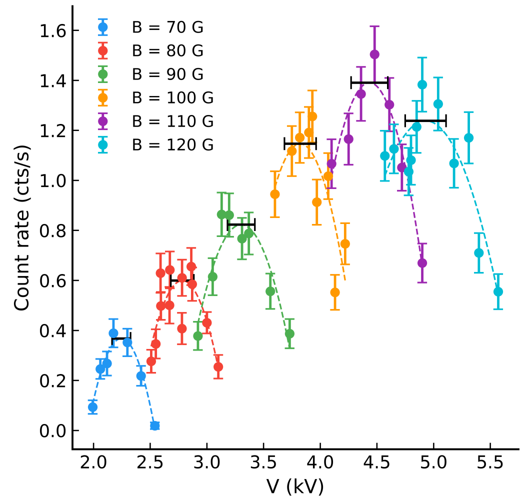

FIG. 9. **Count rate vs. voltage plot shown with theoretical VS width in black bars.** The observed width is greater.

Assume the electrons enter the VS horizontally at speed v. They take time $t = L/v_x$ to reach the PIN diode. In this time, any small transverse force F_y would lead to a transverse displacement of $y = \frac{1}{2}\frac{F_y}{\gamma m}(\frac{L}{v})^2$. To not hit a plate and still be detected, $|y| < d/2$, so $|F_y| < \gamma m d \frac{v^2}{L^2}$. This force is due to $v \neq v^*$, therefore $|F_y| = eB|v - v^*| = \frac{\gamma m v}{\rho}|v - v^*|$. Solving this gives $\frac{\Delta v}{v^*} = \frac{d\rho}{L^2}$, and thus $\frac{\Delta E}{E^*} = \frac{d\rho}{L^2}$. We plot this fractional spread on the same plot as the count rate vs. V plot, and see that the observed spread is larger than the theoretical spread. This suggests that the VS is detecting additional electrons with velocity outside this theoretical acceptance band, likely as the incoming electrons can enter at different angles. The electric field may also be non-uniform due to edge effects. However, we do not expect this to lead to a significant systematic bias in E^* detected as the data points seem symmetric about E^*.

VI. CONCLUSION

Using magnetic momentum selection and a crossed-field VS, we measured the momentum-velocity relation of electrons from radioactive decay and found that they are better described by relativistic than classical dynamics. From this we can also obtain $e/m = (1.31 \pm 0.08_{\text{sys}} \pm 0.06_{\text{ stat}}) \times 10^{11}$ C/kg. Although this is closer to the accepted value than the classical result, it is still low by about $\sim 24\%$, suggesting residual systematic error, most likely in the magnetic field measurement. Overall, the experiment successfully demonstrates the need for relativity in describing energetic electrons.

ACKNOWLEDGMENTS

We thank Liam Sheldon for collaboration on collecting data, and the 8.13 instructors and TAs for their guidance.

[1] J. J. Thomson, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 5, **44**, 293 (1897).
[2] W. Kaufmann, Göttinger Nachrichten , 143 (1901).